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DB-003-001515 Seat No. ________

Third Year B. Sc. (Sem. V) (CBCS) Examination

March – 2022

BSMT-503(A) : Mathematics

(Discrete Mathematics and Complex Analysis-I)
(Theory) (Old Course)

Faculty Code : 003
Subject Code : 001515

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Question. 1 contains 20 short questions of one mark each.
(3) Question. 2 and 3 carry 25 marks each with internal choices.

1 Answer all the questions : **20**

(1) 'R is not reflexive means that R is irreflexive'. (TRUE/FALSE).

(2) If $R = \{,(2,2),(1,3),(2,3),(3,1)\}$ is a relation on $A = \{1,2,3\}$ then find R^{-1}.

(3) Define Symmetric Relation.

(4) Define Complemented lattice.

(5) Define Equivalence relation.

(6) Define Chain.

(7) Define Sub Boolean Algebra.

(8) Define Atom in Boolean Algebra.

(9) Write atoms of $(S_{30}, *, \oplus', 0, 1)$.

(10) Define meet and join in lattice.

(11) Define Complex function.

(12) Define Entire Function.

(13) Define Jordan Arc.

(14) Write Laplace equation.

(15) If L is length of Contour C then $L =$ ________

(16) State Cauchy-Goursat Theorem.

(17) If $C : |z = z_0| = r_0 e^{i\theta}$ then $\int_c \frac{dz}{z - z_0} =$ ________

(18) Define Limit of Complex Variable Functions.

(19) $\lim_{z \to \infty} \frac{2z - 5}{z - 2i}$.

(20) The real part of $f(z) = e^z$.

2 (A) Answer any **three** out of six : **6**

(1) If $(L, *, \oplus ', 0, 1)$ is a bounded lattice then prove that $(1) \alpha * 1 = \alpha (2) \alpha \oplus 1 = 1$

(2) Give an example of a Bounded Lattice which is not Complemented Lattice ?

(3) Draw Hasse Diagram of (S_{30}, D).

(4) In usual notation prove that $xRy \Leftrightarrow [x] = [y]$

(5) In usual notation prove that $A(x') = A - A(x)$.

(6) For Boolean Algebra $(B, *, \oplus ', 0, 1)$ prove that $(a \oplus b) * (b \oplus c) * (c \oplus a) = (a * b) \oplus (b * c) \oplus (b * c) \oplus (c * a)$.

(B) Answer any **three** out of six : **9**

(1) State and Prove lsotonicity Property.

(2) In the lattice if $a \leq b$ and $c \leq d$, then prove that $a * c \leq b * d$ and $a \oplus c \leq b \oplus d$.

(3) Show that similarity of Matrices on the set of $n \times n$ matrices is an equivalence relation.

(4) Let Boolean Algebra $(B,*,\oplus,',0,1)$ and $x_1,x_2 \in B$ then prove that $A(x_1 * x_2) = A(x_1) \cap A(x_2)$

(5) Obtain Cube array representation of Boolean function $f(x_1,x_2,x_3,x_4) = x_1(x_2 + x_3x'_4)$.

(6) Let a and b be two different atoms in B then prove that $a*b=0$.

(C) Answer any **two** out of five : **10**

(1) Prove that every chain is distributive lattice.

(2) Direct product of Two lattice is also a lattice.

(3) State and Prove De' Morgan's Law.

(4) State and Prove Stone Representation Theorem.

(5) If $(L,*,\oplus)$ be lattice then for any $a,b \in L$ prove that $\text{glb}\{a,b\} = a*b$ and $\text{lub}\{a,b\} = a \oplus b$ with respect to the partial ordering R on L.

3 (A) Answer any **three** out of six : **6**

(1) Show that exp $\overline{z}$ is not analytic.

(2) In usual notation prove that ML-Inequality.

(3) If $f = u + iv$ and it complex conjugate $\overline{f} = u - iv$ is analytic then show that f is constant.

(4) State Lioville's Theorem and Fundamental Theorem of Algebra.

(5) Show $f(z) = \overline{z}$ that is not analytic function.

(6) Find value of $\int_C z^2 dz$ where C is part of $y = x^2$ from $z = 0 \; to \; z = 1 + i$.

(B) Answer any **three** out of six : **9**

(1) Is Complex function $f(z) = \frac{1}{z}$ entire ? Justify your answer.

(2) Prove Laplace equation in Polar Form.

(3) Find an analytic function $f(z)=u+iv$ such that, $\operatorname{Im}(f'(z))=6x(2y-1)$ and $f(0)=3-2i$ also find $f(1+i)$.

(4) Prove that $\left|\int_C \frac{1}{z^2}dz\right| < \sqrt{2}$, where C is the line segment joining points $z=2$ and $z=2i$.

(5) State and Prove Cauchy's Inequality.

(6) Find Value of $\int_C \frac{\sin\left(\pi z^2\right)+\cos \pi z^2}{(z+1)(z+2)} dz$ where $C : |z| = 3$.

(C) Answer any **two** out of five : **10**

(1) State and Prove Morera's Theorem.

(2) State and Prove C–R Condition in Polar form.

(3) Show that $\left|\int_C \frac{\log Z}{z^2}dz\right| < \frac{2\pi(\pi+\log R)}{R}$ where $C : |z| = R$ and $R > 1$.

(4) Prove that $u = r^2 \sin 2\theta$ is Harmonic function and find its conjugate.

(5) Prove that the analytic function of constant modulus is also constant in its domain.
